Forecasting the Distribution of Hourly Electricity Spot Prices

- Accounting for Cross Correlation Patterns and Non-Normality of Price Distributions

Arne Vogler
Co-Authors: Christoph Weber, Christian Pape and Oliver Woll

Berlin, 10th February 2017

UNIVERSITÄT
DUISBURG
ESSEN

Open-Minded

Agenda

The Rationale behind Distribution Forecasts

Introduction

- Weron (2014) maintains that, despite being well established in other fields of time series analysis, distribution forecasting has received little attention in electricity price forecasting.
- Yet, increased production of variable RES causes higher uncertainty.
- Thus, the usage of point forecasts only reduces the quality of decision making, due to the reduced amount of information provided.
- Forecasting the distribution of hourly prices is more appropriate for
  - the valuation of assets' flexibilities and optionality,
  - short-term decision making such as dispatch,
  - and providing further information about forecast quality.

An Econometric-Stochastic Approach (I)

Forecasting Approach

- The present econometric-stochastic model combines several established approaches to adequately capture distribution characteristics.

- Panel Data
  - We model the prices of individual hours separately.

- Multiple Regression Analysis
  - We use a linear regression model to account for the deterministic components of prices and to derive the residuals.

- Mapping to Normal Distribution
  - We map the empirical cumulative distribution function of the residuals to a standard normal cumulative distribution to account for non-normality of the price distribution.

An Econometric-Stochastic Approach (II)

UNIVERSITÄT DUISBURG ESSEN
Open-Minded

Forecasting Approach

- Factor Model
  - We apply a factor model to the transformed residual time series to identify common factors and to thereby account for cross correlation between hours.
- ARMA-GARCH Class
  - We model the time series of the identified factors using ARMA-GARCH specifications to account for autocorrelation and time-varying volatility.
- Monte Carlo Simulation
  - We reverse the estimation procedure using Monte Carlo simulations to derive prediction samples.

→ We essentially characterize the distribution of $x_{t,h}$ (the price of hour h at day t) as the empirical cumulative distribution function of a Monte

The Evaluation Framework (I)

Evaluation of Forecast Quality

- Given a sample $\{y_t, I_t\}_{t=1}^{T}$, we seek to test whether $y_t|I_t$ has a specific parametric form.
- Thus, we wish to test the following null hypothesis

$$H_0\colon \Pr(Y_t \leq y|I_t, \theta_0) = F_t(y|I_t, \theta_0)$$

- That is, we seek to assess calibration.
- The evaluation of distribution forecasts rests on the probability integral transform (PIT), also know as Rosenblatt transformation (1952).
  - Under the null hypothesis $F_t(y_t|I_t, \theta_0)$ follows a uniform distribution on [0,1].
  - Additionally, the PIT values from a k-step-ahead forecast should be at most (k-1)-dependent, depending on information set I_t.
  - The PIT values of the distribution forecasts, $F_t\left(y_t\middle|I_t, \hat{\theta}_T\right)$, over a hold-out sample can be used to assess calibration.

The Evaluation Framework (II)

- The graphical evaluation framework
  - The classic econometric testing framework rests on a graphical analysis of these PIT values.
  - Histogram and Sample Autocorrelation Function
  - Yet, it should be noted that (k-1) dependence hinges crucially on I_t being equal to the "relevant" information set.

- Evaluation and formal tests
  - Depending on the information set, the PIT values may exhibit autocorrelation, which formal tests have to account for.
  - Thus, classic Kolmogorov-type tests that rely on i.i.d. observations cannot be applied.
  - Knüppel (2015) proposes a test that is robust to autocorrelation and for which standard critical values can be used.

The Evaluation Framework (III)

- An alternative evaluation framework
  - The probabilistic forecasting test framework rests mainly on the evaluation of the uniformity of the PIT values (graphically and formally), sharpness and various scores measures.
  - The proposed paradigm is to minimize sharpness subject to calibration, where sharpness is a characteristic of the forecast only and refers to the concentration of the distribution forecast.

→ Calibration constitutes a necessary but not sufficient condition for an ideal distribution forecast. We thus require the PIT values to be at least uniformly distributed.

→ Any dependence patterns may shed light on the characteristics of the information set underpinning our specification.

Application (I)

- We test our econometric-stochastic approach against German day-ahead prices for 2014 and 2015 separately

- We consider 12 different specifications.
  - ARMA-GARCH Class: AR(1), AR(2) and ARMA(1,1)-GARCH(1,1)
  - Factor Model: on and off
  - Sample Size: 730 and 184

- We calculate daily out-of-sample day-ahead forecasts using a rolling window for 2014 and 2015; thus, running 8760 Monte Carlo price simulations for each year and specification.

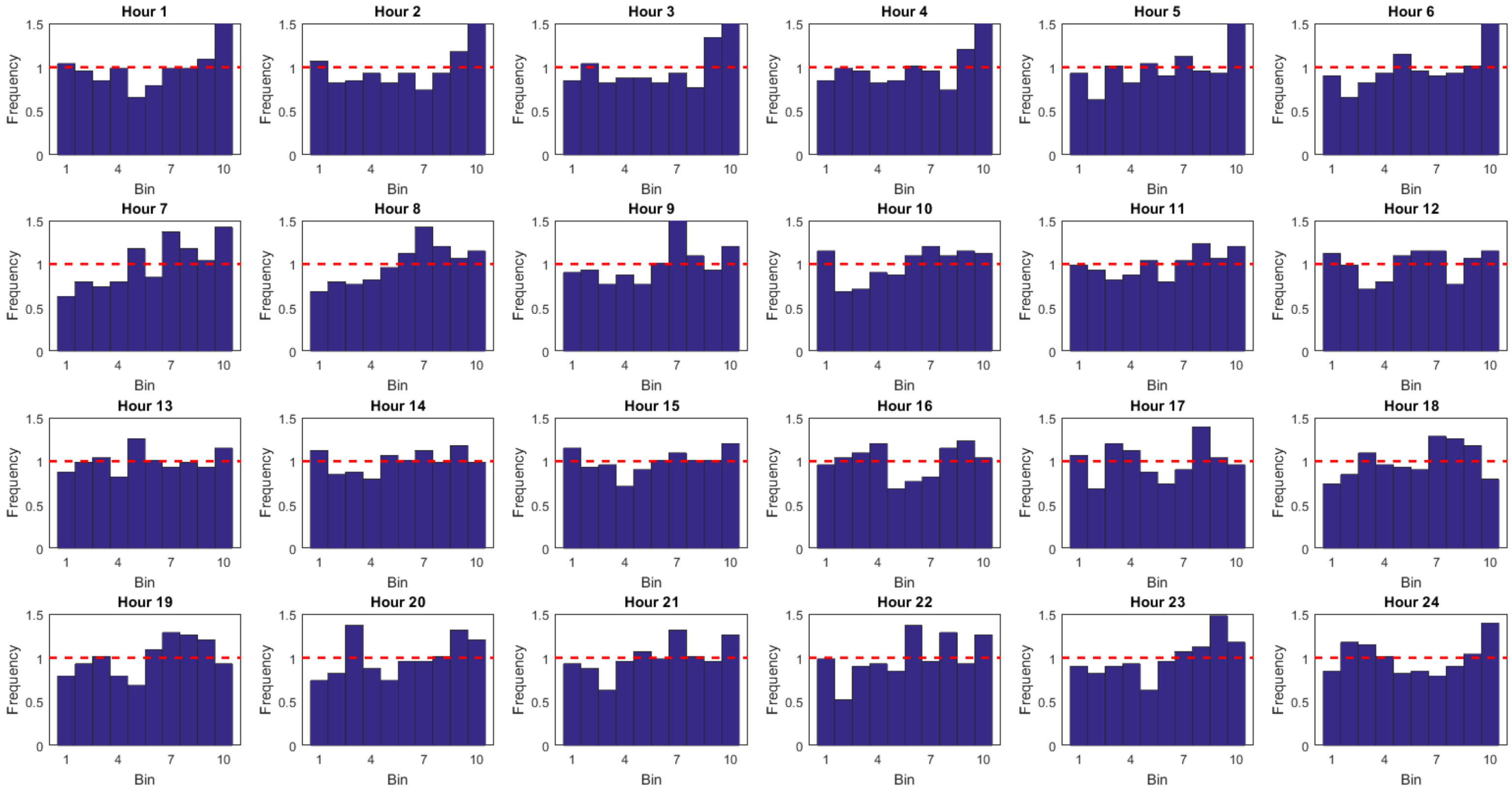
- Based on the evaluation framework, we conclude …
  - … the AR(2) model with the factor model to work best for 2014
  - … the AR(2) model without the factor model to work best for 2015

Application (II)

Application and Results

- We fail to reject the null hypothesis of calibration for 22 hours of 2015 under the preferred specification.

1



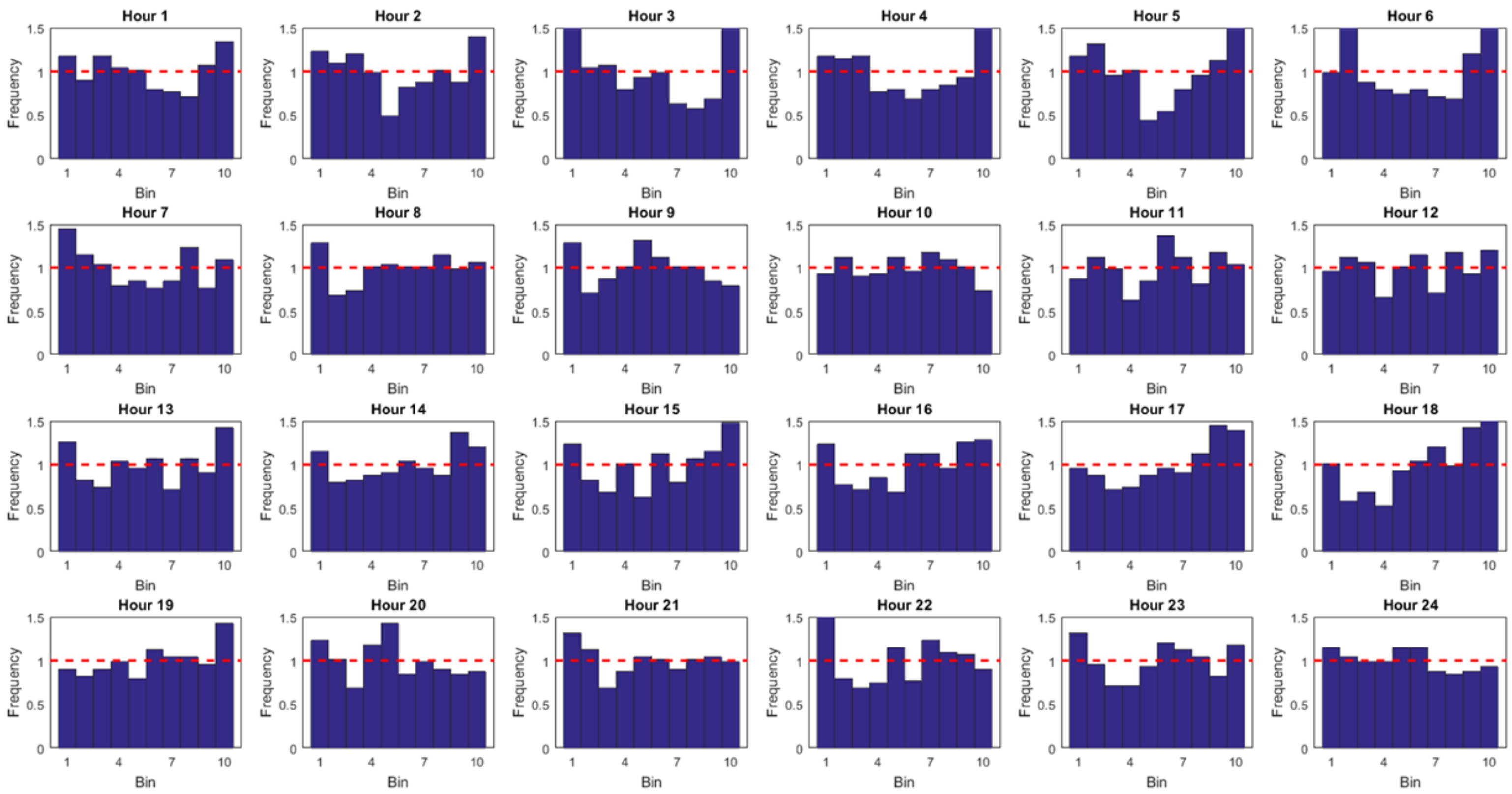
House of Energy Markets & Finance

Application (III)

Application and Results

- We fail to reject the null hypothesis of calibration for 19 hours of 2014 under the preferred specification.

2



Application (IV)

UNIVERSITÄT DUISBURG ESSEN

Open-Minded

Application and Results

- The formal calibration tests, due to Knüppel (2015), confirms the results of the preceding graphical analysis.

Subsample	Method	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	Sum
2014	PCA0_AR2_730	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	6
	PCA1_AR2_730	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	9
	PCA0_AR2_184	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	18
	② PCA1_AR2_184	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	19
2015	① PCA0_AR2_730	1	1	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	22
	PCA1_AR2_730	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	20
	PCA0_AR2_184	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	0	1	1	0	1	20
	PCA1_AR2_184	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	1	0	1	1	1	1	15

!

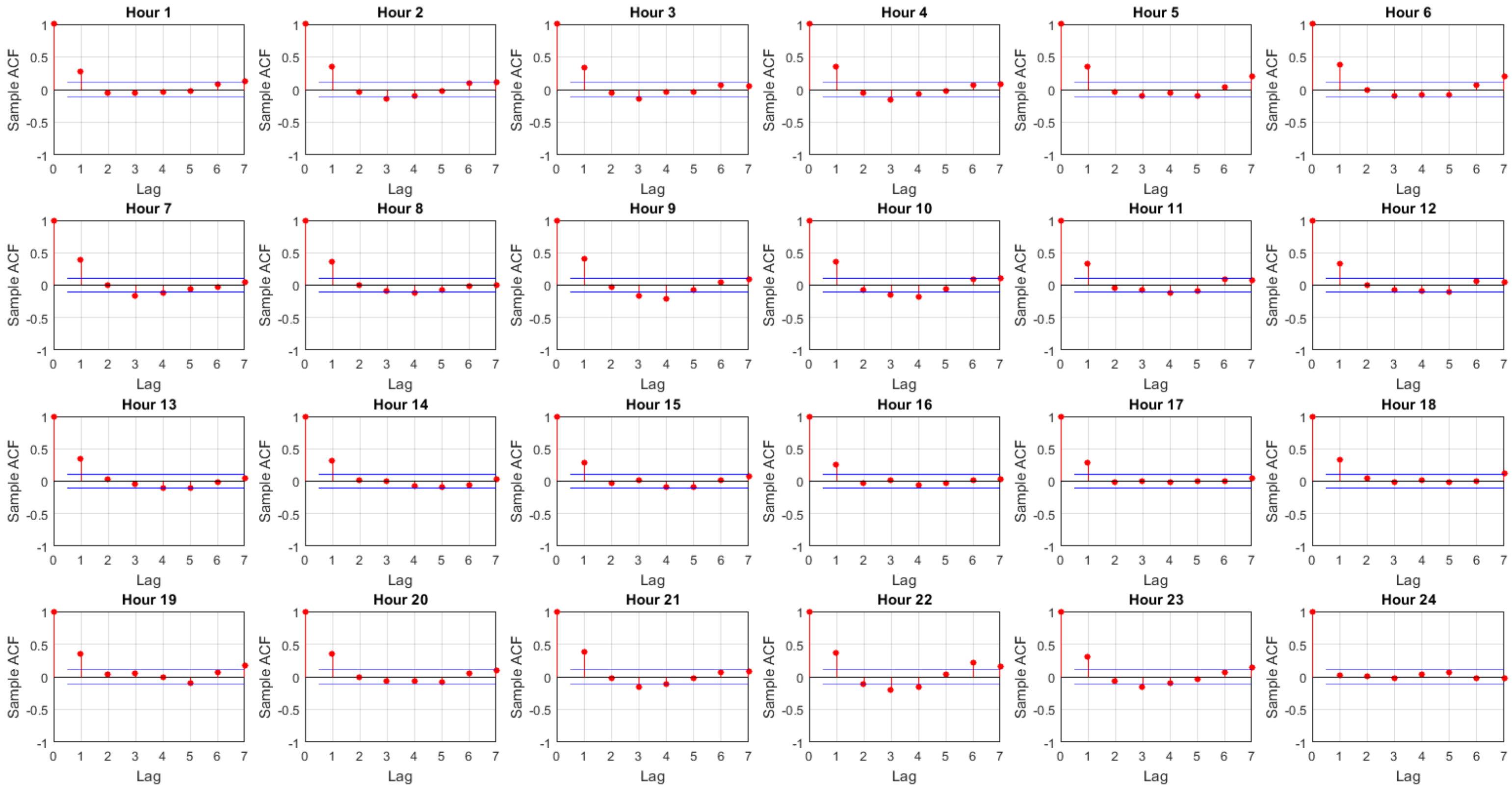
- The present econometric-stochastic approach delivers calibrated distribution forecasts.

House of Energy Markets & Finance

Application (V)

Application and Results

- Yet, the analysis of the sample autocorrelation function uncovers violation of the at most (k-1) dependence criterion for 2015.



House of Energy Markets & Finance

Conclusion

- The econometric-stochastic approach is able to capture the main characteristics of daily hourly prices in Germany and delivers calibrated distribution forecasts.

- A few comments on model particularities are warranted
  - Factor models adequately address cross correlations and ensure smooth price paths
  - Time-varying volatility seems to be less important for price processes of individual hours, as GARCH specifications do not improve results
  - The conditional distributions are correctly specified with respect to the considered information set; yet, dynamic misspecification seems to be present.

Thank you for your attention!

Arne Vogler

House of Energy Markets & Finance
Universität Duisburg-Essen
Weststadttürme | Berliner Platz 6-8
45127 Essen

Estimation and Simulation Procedure (I)

Backup

- (i) Determine the main deterministic drivers and the residuals
  - Regression model to account for deterministic factors

$$x_{t,h} = \beta_0 + \beta_1(L_{t,h} - S_{t,h}) + \beta_2(W_{t,h}) + \beta_3(C_{coal,t}) + \beta_4\,(C_{Gas,t}) + \varepsilon_{t,h}$$

  - β: Regression coefficients
  - $L_{t,h}$: Load
  - $S_{t,h}$: Solar
  - $W_{t,h}$: Wind
  - $C_{Coal,t}$ and $C_{Gas,t}$: typical variabel costs of power plants incl. emission costs
  - $\varepsilon_{t,h}$: Residuals

Estimation and Simulation Procedure (II)

Backup

- (ii) Map the empirical CDF of residuals onto a normal distribution

$$T_h \colon \varepsilon_{t,h} \mapsto \Phi^{-1}\left(C_h\left(\varepsilon_{t,h}\right)\right)$$

  - C_h: Empirical CDF of residuals in hour h
  - Φ: CDF of the standard normal distribution
  - Graphical representation corresponds to Q-Q-plot (Quantile Mapping)

- (iii) Factor Model
  - Normal Residuals

$$u_{t,h} = T_h\left(\varepsilon_{t,h}\right)$$

  - The common factors are constructed using a principal component analysis on the correlation matrix of the transformed factors.

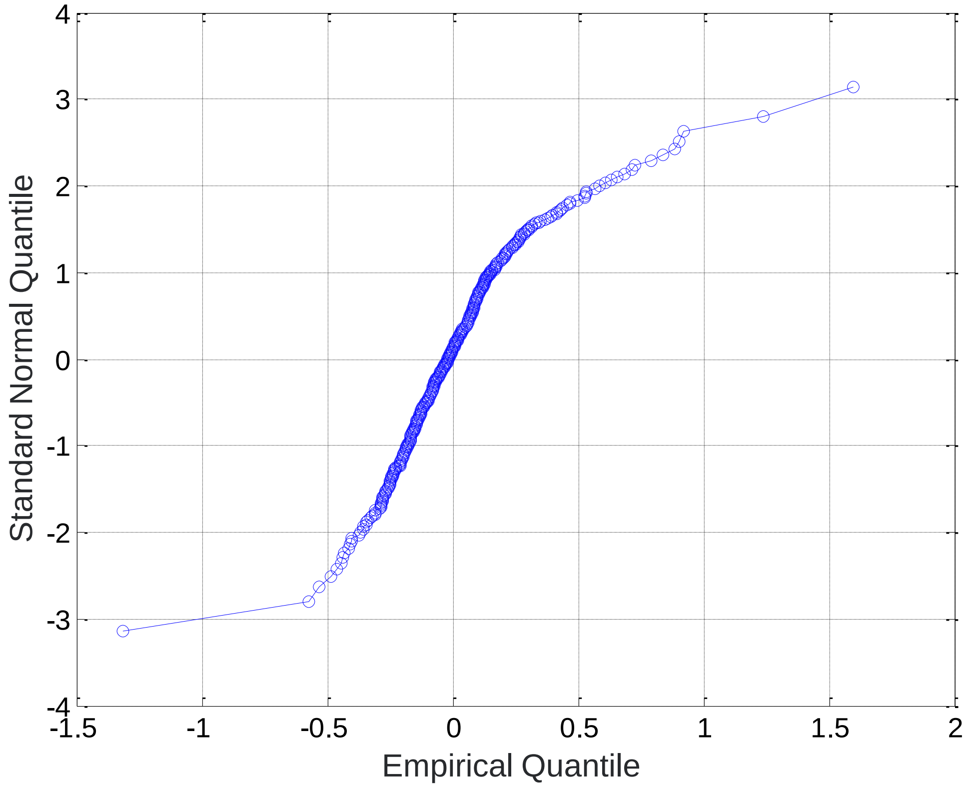
Backup

- (iv) Factor time series are modelled with ARMA-GARCH specifications
  - ARMA (1,1): $f_{t,i} = \alpha_{1,i} f_{t-1,i} + \alpha_{2,i} w_{t-1,i} + w_{t,i}$
    - α: Coefficients for ARMA part
    - $f_{t,i}$: Factors i from different time steps t
    - $w_{t,i}$: Error term from different time steps and $w_{t,i} \sim N(\mu, \boldsymbol{\sigma})$
  - GARCH (1,1): $\sigma^2_{t,i} = \gamma_{0,i} + \gamma_{1,i}\, \sigma^2_{t-1,i} + \gamma_{2,i}\, w^2_{t-1,i}$
    - γ: Coefficients for GARCH part (γ_0 = constant term)
    - $w^2_{t-1,i}$: Error term from the previous time step
    - $\sigma^2_{t-1,i}$: Volatility from the previous time step
- (v) Maximum Likelihood Estimation of parameters using a rolling window
- (vi) Price distributions are simulated by performing the steps (i) to (v) in reverse order

Q-Q-Plot

Backup

- Quantile Mapping



Orthogonal Factor Model

Backup

- The Orthogonal Factor Model (Johnson and Wichern (2002))

$$X = \mu + LF + \varepsilon$$

implies a specific covariance structure

$$\Sigma = LL' + \Psi, \qquad \Psi = Cov(\varepsilon)$$

which can be used to solve for factor loadings L and common factors F by spectral decomposition.

$$\Sigma = \underbrace{\left[\sqrt{\lambda_1} e_1, \ldots, \sqrt{\lambda_p} e_p\right]}_{L} \left[\sqrt{\lambda_1} e_1, \ldots, \sqrt{\lambda_p} e_p\right]'$$