Probabilistic Load Forecasting

Florian Ziel
University of Duisburg-Essen

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Announcement: INREC, 24-25 Sep 2018, Essen

Call-for-Papers for the 7th International Ruhr Energy Conference (INREC)

Uncertainties in Energy Markets

September 24-25, 2018, Essen, Germany

- Keynote speakers:
  - Katja van Doren, RWE Generation SE (GER)
    Political and regulatory uncertainties in the energy markets: an industry perspective
  - Prof. Andreas Löschel, University of Münster (GER)
    Energy Transition in Germany - Status quo and Challenges
  - Prof. Stein-Erik Fleten, NTNU Trondheim (NOR)
    Coordinated vs sequential bidding into short-term electricity markets
  - Prof. Rafał Weron, Wrocław UST (POL)
    Recent advances in electricity price forecasting: A 2018 perspective
- Best paper award (sponsored by GEE)
- Organizers: Prof. Christoph Weber, Prof. Florian Ziel
- www.inrec.wiwi.uni-due.de

Motivation: Point forecasting

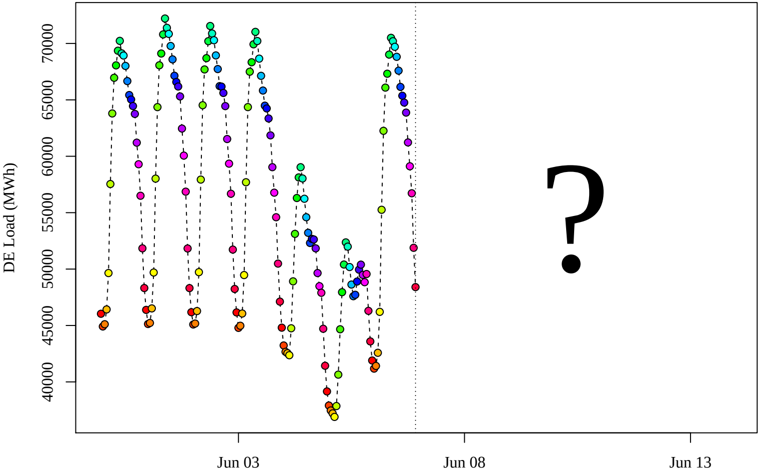
- in energy systems and market modelling various (fundamental) model elements are uncertain
- In particular: electricity load/demand, renewable energy production (esp. wind and solar)

$\rightsquigarrow$ Models with uncertainty are required
if modelling time is in future $\rightsquigarrow$ forecasting models

Standard setting:

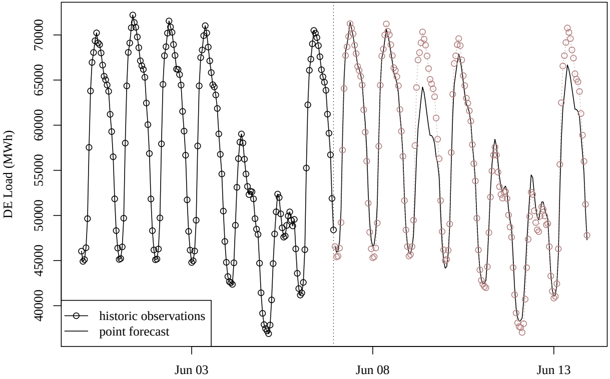
- given historic data (and a model) creating a H-step ahead forecast
- target of interest $\boldsymbol{Y} = (Y_1, \ldots, Y_H) \sim \boldsymbol{F_Y}$, H-dim. random variable
e.g. hourly day-ahead load (24-dim.), hourly weak-ahead load (168-dim.)
- in practice the true distribution $\boldsymbol{F_Y}$ is unknown we just observe $\boldsymbol{y}$
- if we have a *forecast* we can only compare the performance by comparing it with $\boldsymbol{y}$
- evaluation relies on some repeatability of the forecasting experiment

Forecasting Problem

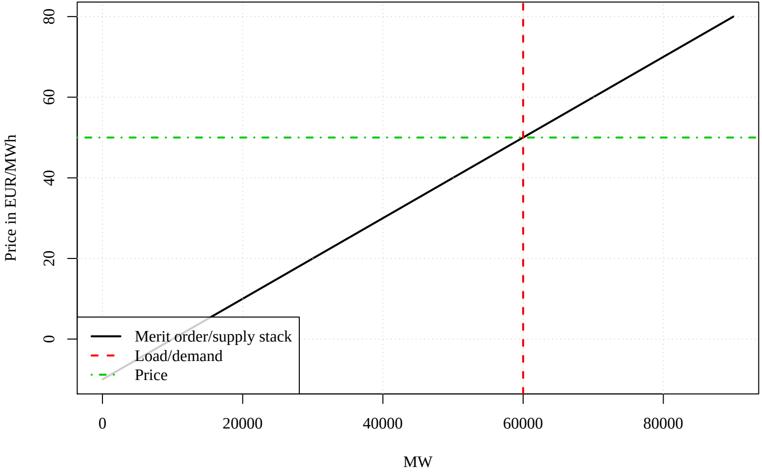


Reporting and evaluating point forecasts

- $\widehat{\boldsymbol{X}}$ estimator for a centre point e.g. $\mathbb{E}(\boldsymbol{Y})$ or $\text{Med}(\boldsymbol{Y})$
- evaluation based on forecasting error $\boldsymbol{Y} - \widehat{\boldsymbol{X}}$, resp. $\boldsymbol{y} - \widehat{\boldsymbol{X}}$
- $\mathbb{E}$ can be strictly proper evaluated using MSE (mean square error)
- Med can be strictly proper evaluated using MAE (mean absolute error)



Why point forecasting is not sufficient?



- deterministic and linear merit order curve
- deterministic and inelastic load/demand curve
- Here: Electricity price 50 EUR/MWh

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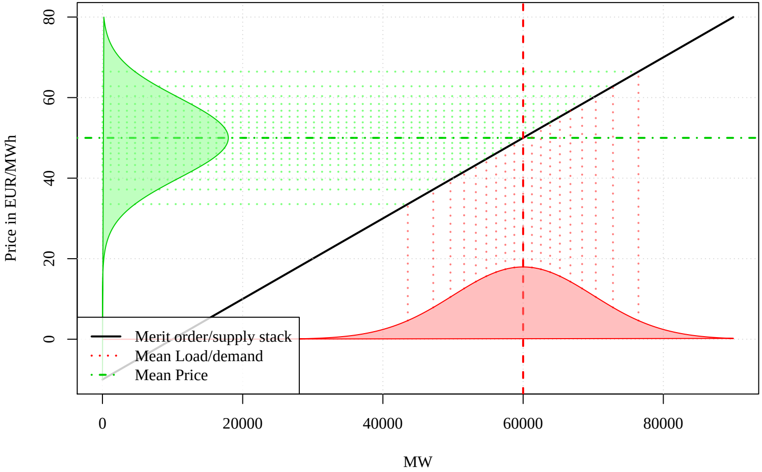
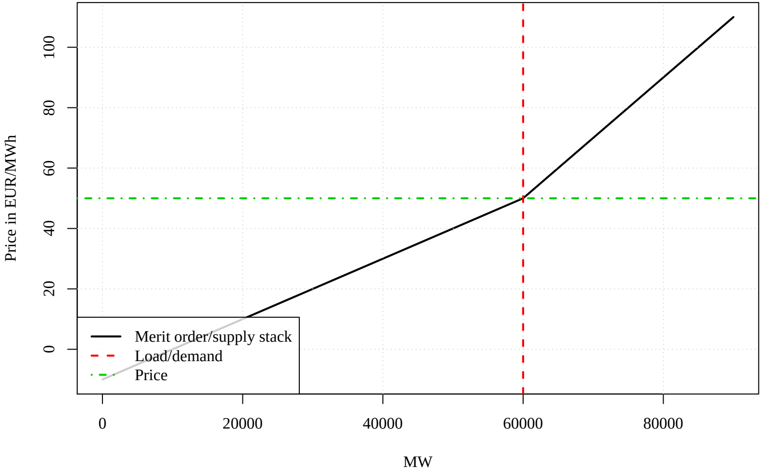


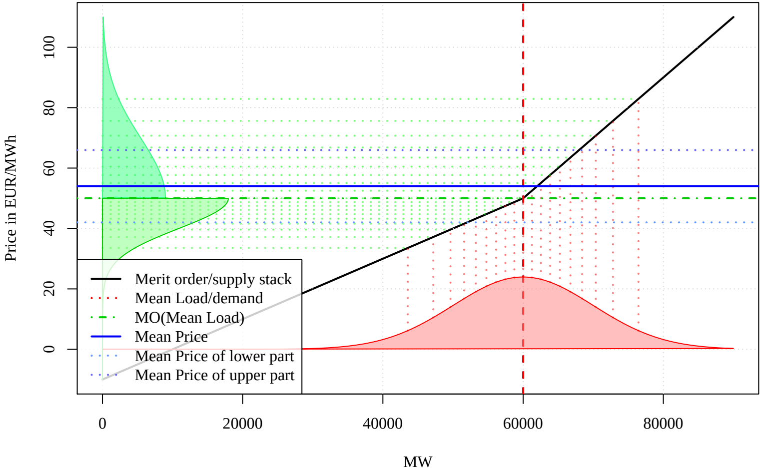
- deterministic and linear merit order curve
- uncertain and inelastic load/demand curve
- Here: Mean electricity price 50 EUR/MWh

Why point forecasting is not sufficient?



- deterministic and non-linear merit order curve
- deterministic and inelastic load/demand curve
- Here: Electricity price 50 EUR/MWh

Why point forecasting is not sufficient?



- deterministic and non-linear merit order curve
- uncertain and inelastic load/demand curve
- Here: Mean electricity price $\approx$ 54 EUR/MWh $\neq$ 50 EUR/MWh

Solution: Probabilistic forecasting

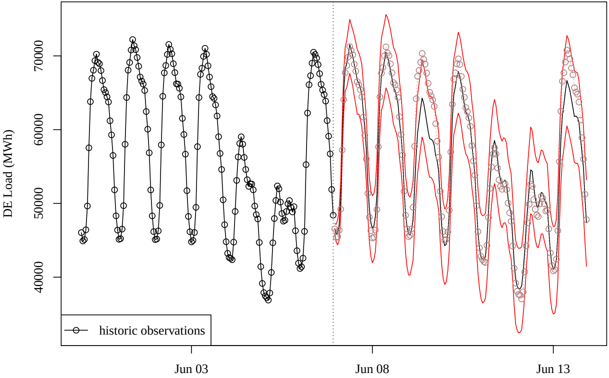
probabilistic forecasting:
Forecast which characterises the uncertainty in the forecast

four main option

- **Prediction intervals**
  e.g. Mean Forecast + Standard deviation ($\widehat{\mu} \pm K\widehat{\sigma}$)
- **Quantile forecasts** on a quantile grid
- **Density forecasts** (strictly marginal densities)
- **Ensemble forecasting**

Probabilistic forecasting

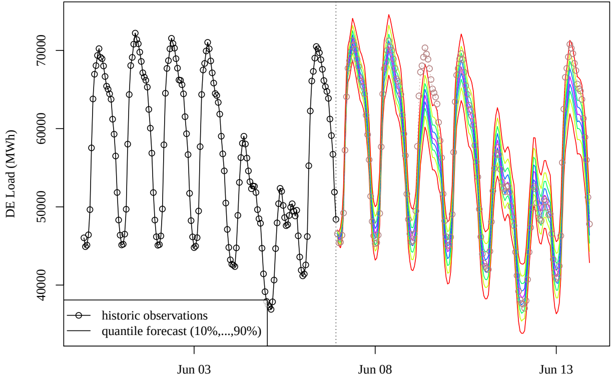
- Prediction intervals
  e.g. Mean + Standard deviation ($\widehat{\mu} \pm K\widehat{\sigma}$)



- Problem: Too simple, uncertainty not fully covered

Probabilistic forecasting: Quantile forecasting

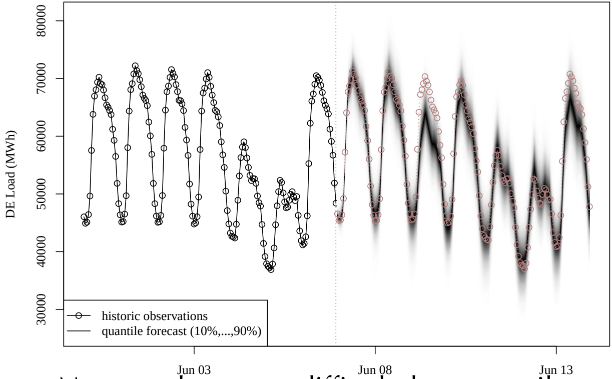
- Quantile Forecasts on a quantile grid (e.g. $10\%, \ldots, 90\%$)



- Relatively popular (e.g. Global Energy Forecasting Competitions (99%-tiles))
- Strictly proper evaluation: pinball score/ quantile loss
- Problem: Still too simple, dependency structure not covered

Probabilistic forecasting: Density forecasting

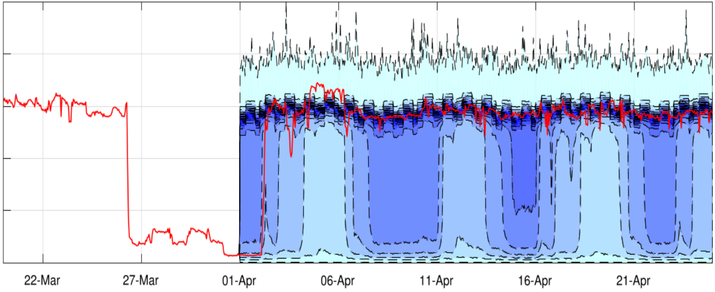
- Density forecasts (strictly marginal densities)



- Not popular, more difficult than quantile approach, but share same properties
- Strictly proper evaluation: continuous rank probability score (CRPS) = limiting case of pinball score
- Problem: Still too simple, dependency structure not covered

Probabilistic forecasting evaluation

- Problem with standard probabilistic methods (e.g quantile forecasting):
  - forecasting only the marginals distributions
  - ignoring the dependency structure
    crucial for industrial load forecasting



(source: Berk, Hoffmann, Müller (2017) *International Journal of Forecasting*)

- require full multivariate forecast $\boldsymbol{F}_{\mathbf{X}}$ for $\boldsymbol{F}_{\mathbf{Y}}$ (multivariate H-dim. density forecast)

Reporting multivariate forecasts

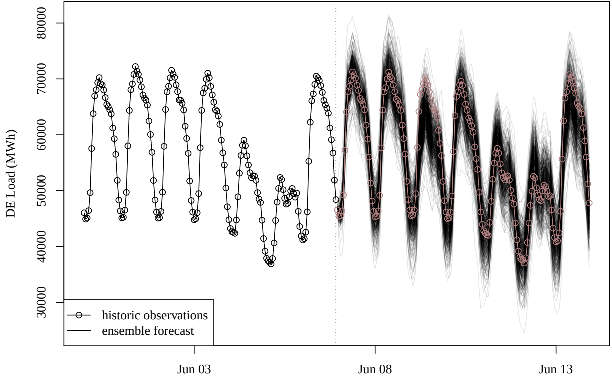
- for sophisticated problems forecast distribution $\boldsymbol{F}_{\boldsymbol{X}}$ (or density $\boldsymbol{f}_{\boldsymbol{X}}$) is not explicitly available.
- reporting forecast as a large ensemble $\boldsymbol{X}^{(1)}, \dots, \boldsymbol{X}^{(M)}$ for forecasting $\boldsymbol{Y}$:
  Ensemble forecasting

Evaluation requires a forecasting study:

- repeat N (similar) forecasting experiments in a rolling window forecasting study: forecasts $\boldsymbol{X}_1, \dots, \boldsymbol{X}_N$ for $\boldsymbol{Y}_1, \dots, \boldsymbol{Y}_N$
- realised ensemble forecasts $\mathcal{X}_i = (\boldsymbol{x}_i^{(1)}, \dots, \boldsymbol{x}_i^{(M)})'$ of the forecasting distribution $\boldsymbol{X}_i$ for $\boldsymbol{Y}_i$

Probabilistic forecasting: Ensemble forecasting

- Ensemble forecasting (Simulating many path from the model)



- Not popular (yet), somehow more difficult to use and requires good models
- No Problems, theoretically all problems can be solved (only possible computational burdens)

Illustration rolling window forecasting study

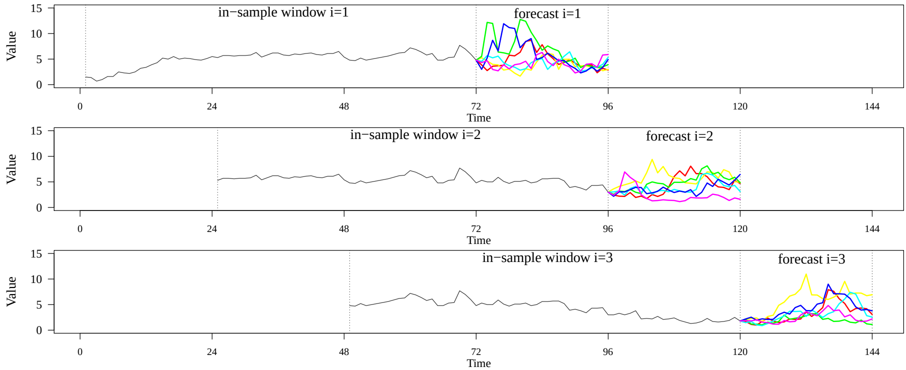


Figure: Illustration of a rolling window forecasting study with non-overlapping windows ($s_i = H(i-1)$) for $i = 1, \ldots, 3$ windows and $M = 6$ forecast samples $\boldsymbol{x}_{T,i}^{(1)}, \ldots, \boldsymbol{x}_{T,i}^{(M)}$ for each window i.

Evaluation measures for multivariate distributions

some measures available

- **Energy score**

$$\text{ES}_\beta(\boldsymbol{F}_{\boldsymbol{X}}, \boldsymbol{y}) = \mathbb{E}\left(\|\boldsymbol{X} - \boldsymbol{y}\|_2^\beta\right) - \frac{1}{2}\mathbb{E}\left(\|\boldsymbol{X} - \widetilde{\boldsymbol{X}}\|_2^\beta\right) \tag{1}$$

  - $\beta > 0$, $\boldsymbol{X}, \widetilde{\boldsymbol{X}} \overset{iid}{\sim} \boldsymbol{F}_{\boldsymbol{X}}$
  - if $H = 1$ and $\beta = 1 \rightsquigarrow$ CRPS
  - strictly proper

- **Variogram score**

$$\text{VS}_p(\boldsymbol{F}_{\boldsymbol{X}}, \boldsymbol{y}; \boldsymbol{W}) = \sum_{i=1}^{H}\sum_{j=1}^{H} w_{i,j}(|y_i - y_j|^p - \mathbb{E}|X_i - X_j|^p)^2$$

  - with $p > 0$ and weight matrix $\boldsymbol{W} = (w_{i,j})_{i,j}$ (usually $w_{i,j} = c$)
  - not strictly proper *(forecasts with shifted mean have same score)*

Evaluation measures for multivariate distributions

- **Log-score**

$$\mathrm{LogS}(\boldsymbol{F_X}, \boldsymbol{y}) = \log(\boldsymbol{f_X}(\boldsymbol{y})).$$

  - where $\boldsymbol{f_X}$ is density of $\boldsymbol{F_X}$
  - strictly proper
  - density forecast for $\boldsymbol{X}$ often not available (even if $\boldsymbol{X}$ is continuous)

- **Dawid-Sebastiani score**

$$\mathrm{DSS}(\boldsymbol{F_X}, \boldsymbol{y}) = \log(|\boldsymbol{\Sigma_X}|) + (\boldsymbol{y} - \boldsymbol{\mu_X})' \boldsymbol{\Sigma_X}^{-1} (\boldsymbol{y} - \boldsymbol{\mu_X})$$

  - with $\boldsymbol{\mu_X}$ and $\boldsymbol{\Sigma_X}$ as mean and covariance matrix of $\boldsymbol{X}$.
  - optimal if $\boldsymbol{Y}$ is normally distributed
  - not strictly proper

- Summary on Scores:
  - only energy score and log-score strictly proper
  - log-score not useful for practice as density forecast is required

Estimating the energy score

- for standard (multivariate) scores estimation straight forward, e.g.

$$\mathrm{ES}_\beta(\boldsymbol{F}_{\boldsymbol{X}}, \boldsymbol{y}) = \mathbb{E}\left(\|\boldsymbol{X} - \boldsymbol{y}\|_2^\beta\right) - \frac{1}{2}\mathbb{E}\left(\|\boldsymbol{X} - \widetilde{\boldsymbol{X}}\|_2^\beta\right)$$

estimated by

$$\widehat{\mathrm{ES}}_\beta^{\text{tri}} = \frac{1}{M}\sum_{j=1}^{M}\left\|\boldsymbol{X}_{T,i}^{(j)} - \boldsymbol{y}_{T,i}\right\|_2^\beta - \frac{1}{M(M-1)}\sum_{j=1}^{M}\sum_{l=j+1}^{M}\left\|\boldsymbol{X}_{T,i}^{(j)} - \boldsymbol{X}_{T,i}^{(l)}\right\|_2^\beta$$

or alternatively by

$$\widehat{\mathrm{ES}}_\beta^{\text{lin}} = \frac{1}{M}\sum_{j=1}^{M}\left\|\boldsymbol{X}_{T,i}^{(j)} - \boldsymbol{y}_{T,i}\right\|_2^\beta - \frac{1}{2M}\sum_{j=1}^{M}\left\|\boldsymbol{X}_{T,i}^{(j)} - \boldsymbol{X}_{T,i}^{(j+1)}\right\|_2^\beta$$

to reduce computational costs.

Ensemble forecasting for electricity load

Requires:

- Recursive time series model: e.g. AR(1)

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \varepsilon_t$$

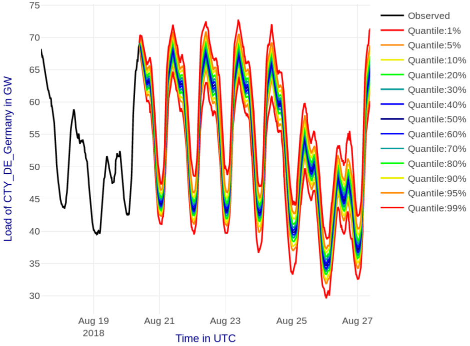
- Recursive forecasting, simulate first Y_{T+1}, then Y_{T+2}, ...
- External regressors (fundamental inputs) need to be forecasted as well
  $\rightsquigarrow$ only useful if we have good forecasts available for the regressor available, e.g.

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 \text{Temperature}_t + \varepsilon_t$$

  $\rightsquigarrow$ deterministic effects are extrmly valuable (esp. seasonal and holiday pattern, (known) maintenance periods)

Summary

- **Non-linear problems under uncertainty require probabilistic forecasts**
- **Inter-temporal problems require ensemble forecast**
- **Ensemble forecasts strictly proper evaluated by energy score**



PS: quantile forecast is reported, model provides ensemble forecasts as well)

www.uee.wiwi.uni-due.de/en/research/load-forecasting/

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