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On forecasting events with multivariate Hawkes processes with time-varying baseline intensity with an application to market order arrivals on the intraday market for power deliveries in Germany

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Motivation

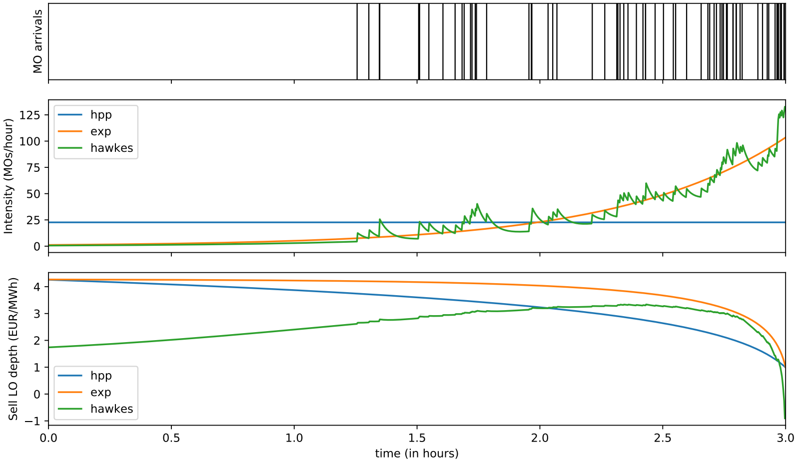

Figure: MO arrivals, intensities and optimal sell LO depths.

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Some literature

- The Hawkes process was introduced in Hawkes (1971) and involves an intensity which has a baseline and which is excited by events of the process.
- In Bowsher (2007) the Hawkes process is used for the first time to model events on financial markets.
- Chen and Hall (2013) is an example for a study where the baseline intensity is allowed to vary deterministically over time.
- In Rambaldi et al. (2017) differences in self-excitement for groups of market orders with different volumes and their interaction are studied.

Multivariate Hawkes process

Consider a d-dimensional point process with associated counting process $(\boldsymbol{N}(t))_{t\geq 0}$. It is a Hawkes process if its intensity $(\boldsymbol{\lambda}(t))_{t\geq 0}$ has the form

$$\boldsymbol{\lambda}(t) = \boldsymbol{\mu}(t) + \int_0^t \phi\,(t-u)\,d\boldsymbol{N}(u),$$

where $\boldsymbol{\mu}(t) : \mathbb{R}_{\geq 0} \to \mathbb{R}^d_{>0}$ is the baseline intensity function and $\phi(t) : \mathbb{R}_{>0} \to \mathbb{R}^{d\times d}_{\geq 0}$ is the excitement function.

Baseline intensity and excitement function

For the baseline intensity we assume the following model:

$$\mu_j(t) = \gamma_j e^{\delta_j t}, \quad j \in \{1, \ldots, d\},$$

where $\gamma > \mathbf{0}, \delta \geq \mathbf{0}$ are d-dimensional vectors. The time-varying baseline intensity causes the Hawkes process which is considered here to be non-stationary.

We assume the excitement function to be of exponential form, i.e.

$$\phi_{jk}(t) = \alpha_{jk} e^{-\beta_{jk} t}, \quad j, k \in \{1, \ldots, d\},$$

where $\alpha \geq \mathbf{0}, \beta > \mathbf{0}$ are $d \times d$-dimensional matrices.

Goodness-of-fit

Let $(\Lambda(t))_{t>0}$ denote the compensator of the point process with associated counting process $\boldsymbol{N}(t)$ and intensity $\lambda(t)$, i.e.

$$\Lambda(t) = \int_0^t \lambda(u)du.$$

By $(\tilde{\boldsymbol{N}}(t))_{t>0}$ we denote a counting process which is a transformation of $\boldsymbol{N}(t)$, specifically

$$\tilde{\boldsymbol{N}}(t) = \boldsymbol{N}(\Lambda^{-1}(t)).$$

The random time change theorem says that the point process with which $\tilde{\boldsymbol{N}}(t)$ is associated is a unit-rate Poisson process.It may be applied in practice by testing whether durations between consecutive transformed arrivals are i.i.d. unit-rate exponentially distributed.

Goodness-of-fit

Delivery start	type	excitements	row-wise	N	N_s	$N_{p_{KS}<0.05}$	$N_{p_{LB}<0.05}$
peak	imp	{}	True	1,092	1,048	699	29
		{noimp}	False	1,092	976	527	19
		{imp}	False	1,092	1,033	25	14
		{noimp,imp}	True	1,092	1,035	26	11
		{noimp,imp}	False	1,092	957	15	11
	noimp	{}	True	1,092	925	754	43
		{noimp}	False	1,092	941	49	15
		{imp}	False	1,092	858	659	32
		{noimp,imp}	True	1,092	946	52	19
		{noimp,imp}	False	1,092	788	31	15

Table: Results of goodness-of-fit tests. Products with delivery start in the peak hours in Q2/2015. N is the number of estimated models. N_s is the number of models which are estimated successfully. $N_{p_{KS}<0.05}$ is the number of successfully estimated models where the null hypothesis of the KS test is rejected at 5% significance level. $N_{p_{LB}<0.05}$ is the number of successfully estimated models where the null hypothesis of the LB test for the first five lags is rejected at 5% significance level.

Out-of-sample forecasting

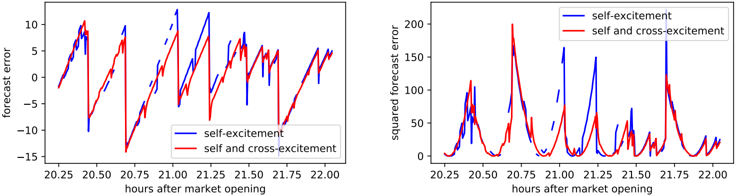
To avoid squared forecast errors being dominated by those errors where the intensity is comparatively low, we suggest forecasting the jumps of $\tilde{\boldsymbol{N}}(t)$ instead.

Let $t_{N(t)+1}$ denote the time of the next jump of some counting process $N(t)$ with intensity $\lambda(t)$ and $\tau_{N(t)+1}$ the transformation of $t_{N(t)+1}$ with the compensator. The optimal forecast of $\tau_{N(t)+1}$ under the quadratic loss function then is

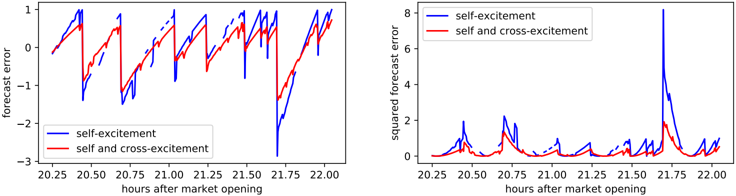
$$\hat{\tau}^*_{N(t)+1} = \Lambda(\hat{t}^*_{N(t)+1}),$$

where $\Lambda(t)$ is the compensator of $\lambda(t)$ and $\hat{t}^*_{N(t)+1}$ is the optimal forecast of $t_{N(t)+1}$ under the quadratic loss function.

Out-of-sample forecasting cont’d



(a) Actual arrival times



(b) Transformed arrival times.

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Some results

					changes to no.							
no.	noimp	imp	row-wise	total	1	2	3	4	5	6	7	8
1	{}	{}	n/a	4	0	0	0	0	0	1	2	1
2	{noimp}	{imp}	n/a	1	0	0	0	0	0	1	0	0
3	{noimp}	{noimp,imp}	True	2	0	0	1	0	0	0	0	1
4	{noimp}	{noimp,imp}	False	1	0	0	0	0	0	0	1	0
5	{noimp,imp}	{imp}	True	5	0	0	0	0	3	0	1	1
6	{noimp,imp}	{imp}	False	1	0	0	0	0	0	1	0	0
7	{noimp,imp}	{noimp,imp}	True	7	0	0	0	0	0	0	7	0
8	{noimp,imp}	{noimp,imp}	False	2	0	0	0	0	0	0	1	1
total				23	0	0	1	0	3	3	12	4

Table: For non-homogeneous Poisson models for the two market order types and for each combination of the models which are promising according to goodness-of-fit testing the number of times the sum of the mean squared errors of the forecasts of actual times of the next events (column “total”) and of transformed times (row “total”) is the smallest. Delivery start is at 12 noon UTC from 2015-04-01 to 2015-04-23.

Conclusion

- Non-homogeneous Poisson process with exponentially increasing intensity does not appear to be a promising model.
- Hawkes process with exponential baseline intensity and exponential excitement function seems to be able capture the dynamics of market order arrivals quite well.
- For the delivery contracts with delivery start at 12:00:00 UTC between 2015-04-01 and 2015-04-23, the forecasts of transformed arrival times from the model with row-wise identical components in β has the smallest MSE most of the times.
- Further research to be done with Diebold-Mariano test.

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